## Reading and Reasoning in Mathematics

Steps to Success



"Mathematics is no more computation than literature is typing."

John Allan Paulos, Ph.D.
 Temple University



#### We need to help students . . .

#### 📕 Formula Sheet

Build their reading skills

Calculator Reference

Type your answer in the box. You may use numbers, a decimal point (.), and/or a negative sign (-) in your answer.

Hartley opened a food truck business to sell food on the street. On day 2, the business earned \$112. On day 5, the business earned \$367. Hartley assumes that the earnings will continue to increase at the same rate. How much will the business earn on day 10?

\$ Build math vocabulary	Depend less on rote memory

Increase conceptual knowledge



#### **Two Essential Strategies**

Helping students learn *how to learn* is critical to aiding the development of higher-order thinking skills

- Modeling is one way to teach students how to learn
- Scaffolding allows students to practice with diminishing support—to build confidence and competence



#### **The Payoff for Students**

- When higher-order thinking skills are used, students carry the knowledge longer.
- Knowledge gained from higher-order thinking processes is more easily transferrable...and that knowledge becomes accessible for solving new problems.





Workbook p. 1

Formative Assessment. Danvers, MA

#### **First Read: Read for Understanding**

- Read through the problem aloud noting your reactions to what you're reading.
- What vocabulary do you not know?
- What's the real-world context of the problem?
- Is there a picture that can help you visualize the problem?
- What questions are being asked?

Miller, P. and Koesling, D. "Mathematics Teaching for Understanding: Reasoning, Reading, and Formative Assessment." Danvers, MA



#### First Read: Read for Understanding



An apartment building contains 12 units consisting of one- and two-bedroom apartments that rent for \$360 and \$450 per month, respectively. When all units are rented, the total monthly rental is \$4,950. What is the number of two-bedroom apartments?



Workbook p. 2

#### **Tiered Vocabulary**

Absolute value	Additive inverse	Algorithm
Attribute	Constant	Distance formula
Exponent	Function	Dependent variable
Independent variable	Linear	Numerical expression
Profit	Property	Proportional gain
Rate of change	Strategy	Value

#### **Building Vocabulary**



Frayer Model – (Barton and Heidema, 2002)

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## **Tools for Building Vocabulary**



Frayer Model – (Barton and Heidema, 2002)



#### Second Read: Identify a Problem-Solving Process

- What is the pertinent information in this problem?
- What problem-solving strategies could I use?
- Which of those problem-solving strategies is best suited for this problem?
- How will I represent the problem in the symbolic language of mathematics?
- What mathematical details will I select as I reason and solve this problem?

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#### Second Read: Identify a Problem-Solving Process



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## **Begin with the Basics**

#### Noticing

- Allows all students to participate
- Work independently or in groups
- What is stated in the problem
- What are the "givens" of the problem

#### Wondering

- Is the planning part
- Talk about strategies to use
- Restate the problem
- Pose questions about what they noticed
- Allows students to slow down and think
- Brainstorm, list, and discuss ideas



## Try It!

lathe	matical	Reasoning -	Candida	ate Name		■ Question 10 of 16
] Ans	wer Exp	lanation 🖯 C	alculator			☐ <u>F</u> lag for Review
A sci park trees shov	entist is . She me ; in the s ; the dat <b>T</b>	studying red r asured the tru ame month ev a for two of th <b>ree 1</b>	naple tre ink diam very othe e trees.	e growth in a eters of a sam r year. The tal <b>Free 2</b>	state iple of bles	Formula Sheet     A Calculator Reference     In year 13, the scientist will put tree wrap around tree 1     to protect it from the winter snow. The height of the tree     wrap needs to be 45 inches.
	Year	Trunk Diameter (inches)	Year	Trunk Diameter (inches)		Tree Wrap
	1	18.6	1	11.4		
	3	19.2	3	12.0		45 inches
	5	19.8	5	12.6		
	7	20.4	7	13.2		
	9	21.0	9	13.8		
	11	21.6	11	14.4		
	13	22.2	13	15.0		square foot, how many square feet of wrap does she
This her o red i	is the fir lata colle naple tre	nal year in whi action is comp ae growth.	ch she wi ete, she	Il collect data. will predict fut	. When ture	need? A. 22 B. 44 C. 121 D. 261

#### What do you notice?

#### What do you wonder?

Workbook p. 5



### It's better to solve one problem five different ways than to solve five different problems.



George Polya, Mathematician
 Stanford University



#### **Must-Have Heuristics**

Workbook p. 6





#### **Strategy 1 - Guess and Check**

Copy the figure below and place the digits 1, 2, 3, 4, and 5 in the circles so that sums across (horizontally) and down (vertically) are the same.



Workbook pp. 7-10



#### **Guess and Check**

Copy the figure below and place the digits 1, 2, 3, 4, and 5 in the circles so that sums across (horizontally) and down (vertically) are the same.

Possible solutions:





#### Strategy 2 - Make a List

Three darts hit this dart board and each scores a 1, 5, or 10. The total score is the sum of the scores for the three darts. There could be three 1's, two 1's and one 5, one 5 and two 10's, and so on. How many different possible total scores could a person get with three darts?





#### Make a List

# of 1's	# of 5's	# of 10's	Score
3	0	0	3
2	1	0	7
2	0	1	12
1	2	0	11
1	1	1	16
1	0	2	21
0	3	0	15
0	2	1	20
0	1	2	25
0	0	3	30





## **Strategy 3 - Draw a Diagram**

In a stock car race, the first five finishers in some order were a Ford, a Pontiac, a Chevrolet, a Buick, and a Dodge.

- The Ford finished seven seconds before the Chevrolet.
- The Pontiac finished six seconds after the Buick.
- The Dodge finished eight seconds after the Buick.
- The Chevrolet finished two seconds before the Pontiac.

In what order did the cars finish the race? What strategy did you use?



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The order is: Ford, Buick, Chevrolet, Pontiac, Dodge.

### **Strategy 4 - Make a Table or Chart**



South Point Amusement Park has a special package for large groups: a flat fee of \$20 and \$6 per person.

If a club has \$100 to spend on admission, what is the largest number of people who can attend?



#### Make a Table or Chart

South Point Amusement Park has a special package for large groups: a flat fee of \$20 and \$6 per person.

If a club has \$100 to spend on admission, what is the largest number of people who can attend?

# of people	Cost X \$6	+ \$20	Total fee	Result
10	60	20	80	Too low
15	90	20	110	Too high
13	78	20	98	Too low
14	84	20	104	Too high

Answer: At most, 13 people can attend for \$100 and they will have \$2 left over.



#### **Strategy 5 - Find a Pattern**

Continue these numerical sequences by finding the next three numbers for each group.



#### **Find a Pattern**

Continue these numerical sequences by finding the next three numbers for each group.

1, 4, 7, 10, 13, \_\_\_\_, \_\_\_\_ (add 3 to the previous term)

#### 1, 4, 7, 10, 13, 16, 19, 22

19, 20, 22, 25, 29, \_\_\_\_, \_\_\_\_, \_\_\_\_ ( add 1 to the previous term, then add 2 to that term, then add three to that term)

#### 19, 20, 22, 25, 29, 34, 40, 47

2, 6, 18, 54, \_\_\_\_, \_\_\_\_ (multiply the previous term by 3 to generate the next term)

#### 2, 6, 18, 54, 162, 486, 1458



#### **Strategy 6 - Make it Simpler**

The houses on Main Street are numbered consecutively from 1 to 150.

How many house numbers contain at least one digit 7?



### Make it Simpler

The houses on Main Street are numbered consecutively from 1 to 150.

How many house numbers contain at least one digit 7?

Break the problem down. First determine how many houses have a seven in the units place. (15)

7, 17, 27, 37, 47, 57, 67, 77, 87, 97, 107, 117, 127, 137, 147

Next, determine how many houses have a seven in tens place. (10)

70, 71, 72, 73, 74, 75, 76, 77, 78, 79

Take out any duplicates (1)

Answer – 24 houses contain at least one digit 7.



# Strategy 7 - Act It Out or Use Objects

The figure shows twelve toothpicks arranged to form three squares. How can you form five squares by moving only three toothpicks?





### Act It Out or Use Objects

The figure shows twelve toothpicks arranged to form three squares. How can you form five squares by moving only three toothpicks?

Answer: One of the squares is formed by the outer boundary of the arrangement. There was no requirement that each of the five squares must be congruent to each of the others.







#### **Strategy 8 - Work Backwards**



Brady was trying to decide when to get up in the morning. He needs 45 minutes to get ready for the workshop he plans to attend. It takes him 25 minutes to drive to the adult center where the workshop will be held. He wanted to get to the center 20 minutes early to stop by his classroom and pick up some materials. If the session starts at 7:30 a.m., what time should he get up, if he wants to give himself 10 extra minutes in case the traffic is bad?



#### **Work Backwards**

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Start with the time he has to be at the workshop -7:30

He needed 20 minutes to go by the classroom - 7:10

He drove for 25 minutes – 6:45

He needed 45 minutes to get ready – 6:00

To allow for 10 minutes in case the traffic was bad – 5:50



## **Strategy 9 - Brainstorm and Write an Equation**

Two apples weigh the same as a banana and a cherry. A banana weighs the same as nine cherries. How many cherries weigh the same as one apple?





#### **Brainstorm and Write an Equation**

Two apples weigh the same as a banana and a cherry. A banana weighs the same as nine cherries. How many cherries weigh the same as one apple?

You will need to use three variables.

- A = the weight of an apple
- B = the weight of a banana
- C = the weight of a cherry

2A = B + C B = 9CSubstituting: 2A = 9C + C2A = 10CA = 5C

Answer: 5 cherries weigh the same as 1 apple



## Strategy 10 – Use Logical Reasoning

Three apples and two pears cost 78 cents. However, two apples and three pears cost 82 cents. What is the total cost of one apple and one pear?

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By combining the two clues given, one can conclude that five apples and five pears cost 78 plus 82 cents, or 160 cents. Divide that by five and you can conclude that one apple and one pear costs 32 cents.

Remember – you are not looking for the cost of an apple or a pear, but the combined cost of both.



#### Third Read: Solve the Problem and Check for Reasonableness

- Now that I understand the problem's content, how can I best use my math skills to solve the problem?
- Am I answering the right question?
- How should the answer to the question be expressed?

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#### **Goals and Givens**

Active Reading Strategy for Problem Solving

- Read problem closely
- Identify the goal the task(s) to be completed
  - Paraphrase what author wants to be done
  - Write in own words
- Identify the givens information relevant to solving the task
  - Look for key terms



#### **Goals and Givens Template**

• • •	and the second	
Goal: What is the question?	Givens: Important details / information that is provided	
Plan: What strategies will you use? May have multiple         Draw and Label Diagram/Picture         Look for special cases         Create and solve a simpler problem	checked. Circle the one that was most effective. patterns  Write an equation able Work backwards e to a similar S you have solved	
Conjecture: (reasonable guess) Predict your answer and any reasoning that results	in your predicted answer	
Solution: (make no assumptions, label everything) <ul> <li>Start Solving using your plan from</li> <li>Move vertically with your work</li> <li>Show/document ALL work</li> <li>Do NOT make any assumpti</li> <li>Continue to refer to the word</li> <li>Be precise and label units was a final answer</li> <li>Persevere! Keep trying event</li> </ul>	i above ions rd problem rhile solving & when you have reached n if it means that you need to change	
your plan. (trying a new stra 。 Go as far as you can!	ategy)	

A	
Mrite a	(complete sentence answering the goal. Include proper units, notation, etc.)
write a	complete sentence that answers your goal with appropriate units.
Verificati	ON: (Explain why your answer makes sense, why it is reasonable, did you answer the goal, is there anoth
Verificati	ON: (Explain why your answer makes sense, why it is reasonable, did you answer the goal, is there anoth it proves your answer is correct?)
Verificati strategy th How do	On: (Explain why your answer makes sense, why it is reasonable, did you answer the goal, is there anot? It proves your answer is correct?) you know your answer is correct?
Verificati strategy the How do Is your a	on: (Explain why your answer makes sense, why it is reasonable, did you answer the goal, is there anot it proves your answer is correct?) you know your answer is correct? nswer reasonable?
Verificati strategy th How do Is your a Did your	On: (Explain why your answer makes sense, why it is reasonable, did you answer the goal, is there anoth it proves your answer is correct?) you know your answer is correct? nswer reasonable? answer the goal?
Verificati strategy th How do Is your a Did your Can you	On: (Explain why your answer makes sense, why it is reasonable, did you answer the goal, is there anot at proves your answer is correct?) you know your answer is correct? nswer reasonable? • answer the goal? convince someone using another method?
Verificati strategy th How do Is your a Did you Can you	On: (Explain why your answer makes sense, why it is reasonable, did you answer the goal, is there anot at proves your answer is correct?) you know your answer is correct? nswer reasonable? • answer the goal? convince someone using another method?

2 pts

1 pt

1 pt

1 pt

3 pts

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## Thank you!

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